

# Linear inequalities

## A LEVEL LINKS

Scheme of work: 1d. Inequalities – linear and quadratic (including graphical solutions)

## Key points

- Solving linear inequalities uses similar methods to those for solving linear equations.
- When you multiply or divide an inequality by a negative number you need to reverse the inequality sign, e.g.  $<$  becomes  $>$ .

## Examples

**Example 1** Solve  $-8 \leq 4x < 16$

$\begin{aligned} -8 &\leq 4x < 16 \\ -2 &\leq x < 4 \end{aligned}$	Divide all three terms by 4.
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**Example 2** Solve  $4 \leq 5x < 10$

$\begin{aligned} 4 &\leq 5x < 10 \\ \frac{4}{5} &\leq x < 2 \end{aligned}$	Divide all three terms by 5.
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**Example 3** Solve  $2x - 5 < 7$

$\begin{aligned} 2x - 5 &< 7 \\ 2x &< 12 \\ x &< 6 \end{aligned}$	<ol style="list-style-type: none"> <li>1 Add 5 to both sides.</li> <li>2 Divide both sides by 2.</li> </ol>
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**Example 4** Solve  $2 - 5x \geq -8$

$\begin{aligned} 2 - 5x &\geq -8 \\ -5x &\geq -10 \\ x &\leq 2 \end{aligned}$	<ol style="list-style-type: none"> <li>1 Subtract 2 from both sides.</li> <li>2 Divide both sides by <math>-5</math>. Remember to reverse the inequality when dividing by a negative number.</li> </ol>
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**Example 5** Solve  $4(x - 2) > 3(9 - x)$

$\begin{aligned} 4(x - 2) &> 3(9 - x) \\ 4x - 8 &> 27 - 3x \\ 7x - 8 &> 27 \\ 7x &> 35 \\ x &> 5 \end{aligned}$	<ol style="list-style-type: none"> <li>1 Expand the brackets.</li> <li>2 Add <math>3x</math> to both sides.</li> <li>3 Add 8 to both sides.</li> <li>4 Divide both sides by 7.</li> </ol>
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## Practice

1 Solve these inequalities.

**a**  $4x > 16$

**b**  $5x - 7 \leq 3$

**c**  $1 \geq 3x + 4$

**d**  $5 - 2x < 12$

**e**  $\frac{x}{2} \geq 5$

**f**  $8 < 3 - \frac{x}{3}$

2 Solve these inequalities.

**a**  $\frac{x}{5} < -4$

**b**  $10 \geq 2x + 3$

**c**  $7 - 3x > -5$

3 Solve

**a**  $2 - 4x \geq 18$

**b**  $3 \leq 7x + 10 < 45$

**c**  $6 - 2x \geq 4$

**d**  $4x + 17 < 2 - x$

**e**  $4 - 5x < -3x$

**f**  $-4x \geq 24$

4 Solve these inequalities.

**a**  $3t + 1 < t + 6$

**b**  $2(3n - 1) \geq n + 5$

5 Solve.

**a**  $3(2 - x) > 2(4 - x) + 4$

**b**  $5(4 - x) > 3(5 - x) + 2$

## Extend

6 Find the set of values of  $x$  for which  $2x + 1 > 11$  and  $4x - 2 > 16 - 2x$ .

**Answers**

**1 a**  $x > 4$

**b**  $x \leq 2$

**c**  $x \leq -1$

**d**  $x > -\frac{7}{2}$

**e**  $x \geq 10$

**f**  $x < -15$

**2 a**  $x < -20$

**b**  $x \leq 3.5$

**c**  $x < 4$

**3 a**  $x \leq -4$

**b**  $-1 \leq x < 5$

**c**  $x \leq 1$

**d**  $x < -3$

**e**  $x > 2$

**f**  $x \leq -6$

**4 a**  $t < \frac{5}{2}$

**b**  $n \geq \frac{7}{5}$

**5 a**  $x < -6$

**b**  $x < \frac{3}{2}$

**6**  $x > 5$  (which also satisfies  $x > 3$ )